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Application of the Cercignani–Lampis scattering kernel to calculations of rarefied gas flows. III. Poiseuille flow and thermal creep through a long tube

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Abstract

The Cercignani–Lampis scattering kernel of the gas–surface interaction is applied to numerical calculations of the Poiseuille flow and thermal creep through a long tube. The S model of the Boltzmann equation was numerically solved by the discrete velocity method. The calculations have been carried out in the wide ranges of the rarefaction parameter and of the accommodation coefficients. Comparing the present results with experimental data the values of the accommodation coefficients have been calculated.

Keywords: Rarefied gas; Gas–surface interaction; Accommodation coefficients; Poiseuille flow; Thermal transpiration

1. Introduction

The gas–surface interaction law in general form reads [1]

$$|v_n|\, f(\mathbf{v}) = \int\limits_{v_n'<0} |v_n'|\, R(\mathbf{v}' \to \mathbf{v})\, f(\mathbf{v}')\, \mathrm{d}\mathbf{v}', \tag{1}$$

where $f(\mathbf{v})$ is the velocity distribution function, $R(\mathbf{v}' \to \mathbf{v})$ is the scattering kernel, $\mathbf{v}'$ and $\mathbf{v}$ are molecular velocities of the incident and reflected particles, respectively. Such a relation is necessary to calculate rarefied gas flows applying the kinetic Boltzmann equation.

The diffuse-specular scattering kernel

$$R(\mathbf{v}' \to \mathbf{v}) = (1-\varepsilon)\delta(\mathbf{v}' - \mathbf{v} + 2\mathbf{n}v_n) + \varepsilon \frac{m^2 v_n}{2\pi (kT_w)^2} \exp\left(-\frac{mv^2}{2kT_w}\right) \tag{2}$$

proposed by Maxwell is widely used in many practical calculations [2]. Here, v_n is the normal component of molecular velocity $\mathbf{v}$ directed into the gas, m is the molecular mass of the gas, T_w is the surface temperature, k is the Boltzmann constant and the parameter ε is the so-called accommodation coefficient. As was noted in the previous papers [3,4], calculations of the parameter ε from different experimental data, such as Poiseuille flow, slip coefficient, thermal creep etc., give quite different values of ε. So, the Maxwell model (2) cannot correctly describe the gas–surface interaction. Moreover, it cannot explain the deviation of the Thermomolecular Pressure Difference (TPD) exponent γ from the value $1/2$ in the free molecular regime,

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while several experimental works, see, e.g., [5,6], showed that the exponent γ varies from 0.4 to 0.5 at low pressure. At the same time, the kernel proposed by Cercignani and Lampis [7] provides more physical description of many transport phenomena in gases including the TPD. This kernel reads

$$R(\mathbf{v}' \to \mathbf{v}) = \frac{m^2 v_n}{2\pi\alpha_n\alpha_t(2-\alpha_t)(kT_w)^2} \exp\left\{-\frac{m[v_n^2+(1-\alpha_n)v_n'^2]}{2kT_w\alpha_n} - \frac{m[\mathbf{v}_t-(1-\alpha_t)\mathbf{v}_t']^2}{2kT_w\alpha_t(2-\alpha_t)}\right\} I_0\left(\frac{\sqrt{1-\alpha_n}\,m v_n v_n'}{\alpha_n kT_w}\right), \tag{3}$$

where

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\phi)\,\mathrm{d}\phi, \tag{4}$$

$\mathbf{v}_t$ is the two-dimensional vector of the tangential velocity. Unlike the Maxwell model (2), the kernel (3) contains the two parameters: the first of them α_t is the accommodation coefficient of tangential momentum and the second one α_n is the accommodation coefficient of kinetic energy due to the normal velocity v_n. The momentum accommodation coefficient α_t varies in the range from 0 to 2. Physically this means that the Cercignani–Lampis (CL) kernel (3) admits the back reflection, which may occur on a rough surface. In the limit case at $\alpha_t = 2$ and $\alpha_n = 0$ a particle changes the sign of its own velocity after a collision with a surface and then, it goes exactly in the back direction. In the case $\alpha_t = 1$ and $\alpha_n = 1$ the kernel (3) provides the diffuse reflection (perfect accommodation), while the values $\alpha_t = 0$ and $\alpha_n = 0$ give the specular reflection. However, for intermediate values of α_t and α_n the CL kernel (3) quite differs from the diffuse-specular one (2).

The aim of the present paper is to apply the CL kernel to calculations of rarefied gas flow through a long tube caused by both pressure and temperature gradients. These data allow us to calculate the TPD exponent too. Then, comparing these results with experimental data available in the open literature, the accommodation coefficients α_t and α_n are calculated for some gas–surface pairs.

2. Input equation

Consider a monoatomic rarefied gas flowing through a long tube due to small longitudinal gradients of pressure P and temperature T denoted as

$$\xi_{\mathrm{P}} = \frac{a}{P}\frac{\mathrm{d}P}{\mathrm{d}x'}, \qquad \xi_{\mathrm{T}} = \frac{a}{T}\frac{\mathrm{d}T}{\mathrm{d}x'}, \tag{5}$$

respectively. Here, x' is the longitudinal coordinate coinciding with the tube axis, a is the tube radius. We assume that the tube is so long that the end effects can be neglected and the flow is considered to be one-dimensional, i.e., the bulk velocity and the heat flux have the longitudinal component only. We are going to calculate the mass flow rate through a cross section of the tube in the whole range of the gas rarefaction defined as

$$\delta = \frac{aP}{\mu}\left(\frac{m}{2kT_0}\right)^{1/2}, \tag{6}$$

where T_0 is the equilibrium temperature and μ is the stress viscosity proportional to the molecular mean free path λ, i.e., the rarefaction parameter δ is inversely proportional to the Knudsen number $\mathrm{Kn} = \lambda/a$. So, in the limit $\delta = 0$ we have the free molecular (collisionless) flow and in the other limit $\delta \to \infty$ the flow regime is hydrodynamic.

Comparing the results of the two previous papers [3,4] based on the S model proposed by Shakhov [8] with those obtained in the work [9,10] from the Boltzmann equation we may conclude that the S model provides reliable numerical results for non-isothermal rarefied gas flows with modest computational efforts. So, like the works [3,4] here we apply the S model too.

For further derivations it is convenient to introduce the following dimensionless quantities

$$x = \frac{x'}{a}, \qquad y = \frac{y'}{a}, \qquad z = \frac{z'}{a}, \tag{7}$$

$$\mathbf{c} = \beta\mathbf{v}, \qquad u = \beta u_x', \qquad q = \frac{\beta}{P_0}q_x', \qquad \beta = \left(\frac{m}{2kT_0}\right)^{1/2}, \tag{8}$$

where x', y', and z' are Cartesian coordinates, u_x' is the longitudinal component of the bulk velocity, q_x' is the longitudinal component of the heat flux, and P_0 is the equilibrium pressure.

Since we assume the pressure and temperature gradients to be small, i.e., $\xi_P \ll 1$ and $\xi_T \ll 1$, the distribution function $f(x, y, z, \mathbf{c})$ is linearized as

$$f(x, y, z, \mathbf{c}) = f^0\left[1 + h(y, z, \mathbf{c}) + \xi_P x + \left(c^2 - \frac{5}{2}\right)\xi_T x\right], \tag{9}$$

$$f^0 = n_0\left(\frac{m}{2\pi k T_0}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_0}\right),$$

where $n_0 = P_0/kT_0$ is an equilibrium number density. Taking into account the axial symmetry of the flow, the linearized S model equation can be written in terms of the dimensionless variables as

$$c_r\frac{\partial h}{\partial r} - \frac{c_\theta}{r}\frac{\partial h}{\partial \theta} = \delta\left[2uc_x + \frac{4}{15}qc_x\left(c^2 - \frac{5}{2}\right) - h\right] - c_x\left[\xi_P + \xi_T\left(c^2 - \frac{5}{2}\right)\right], \tag{10}$$

where c_r and c_θ are radial and azimuth components of the molecular velocity $\mathbf{c}$, respectively, $\theta = \arctan(c_\theta/c_r)$, and $r = \sqrt{y^2 + z^2}$ is the radial coordinate. The dimensionless velocity u and heat flux q are related to the perturbation function as

$$u(r) = \frac{1}{\pi^{3/2}}\int \exp\bigl(-c^2\bigr)h(r, \mathbf{c}), c_x \,\mathrm{d}\mathbf{c}, \tag{11}$$

$$q(r) = \frac{1}{\pi^{3/2}}\int \exp\bigl(-c^2\bigr)h(r, \mathbf{c})c_x\left(c^2 - \frac{5}{2}\right)\mathrm{d}\mathbf{c}. \tag{12}$$

Note, because of the axial symmetry the moments u and q depend only on the radial coordinate r.

Since Eq. (10) is linear, its solution h and the moments u and q can be decomposed into two parts as

$$h = h_P\xi_P + h_T\xi_T, \qquad u = u_P\xi_P + u_T\xi_T, \qquad q = q_P\xi_P + q_T\xi_T. \tag{13}$$

Then, we introduce the two reduced flow rates as

$$G_P = -4\int_0^1 u_P(r)r\,\mathrm{d}r, \qquad G_T = 4\int_0^1 u_T(r)r\,\mathrm{d}r. \tag{14}$$

The first coefficient G_P corresponds to the mass flow due to the pressure gradient and it is called as Poiseuille flow. The second coefficient G_T describes the mass flow rate caused by the temperature gradient and it is called as thermal creep.

It should be noted that here, we assume the small pressure and temperature gradient. However, the numerical data on the coefficients G_P and G_T can be used to calculate the mass flow rate for arbitrary pressure and temperature drops applying the method described in the papers [2,11–13].

The TPD phenomenon mentioned above is as follows. Let us consider a closed system consisting of two reservoirs connected by a long tube. If a temperature ratio T_1/T_2 is maintained between the reservoirs, then a pressure ratio P_1/P_2 between them is established. The relation of P_1/P_2 to T_1/T_2 in general form can be written as

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^\gamma. \tag{15}$$

If the temperature drop between the reservoirs is small then the exponent γ is related to the coefficients G_P and G_T as

$$\gamma = \frac{G_T}{G_P}. \tag{16}$$

If the temperature drop is large, then the exponent γ is calculated by the method described in the work [12].

Substituting (9) into (1) and considering that surface temperature T_w is equal to the equilibrium temperature T_0 in the section $x = 0$ we relate the perturbation of incident particles h^+ to the perturbation of reflected ones h^- as

$$h^+ = \hat{A}_x\hat{A}_\varphi\hat{A}_r h^-, \tag{17}$$

where

$$\hat{A}_i\phi = \int_{-\infty}^{\infty} \exp\bigl(c_x^2 - c_x'^2\bigr)R_i(c_i' \to c_i)\phi(c_i')\,\mathrm{d}c_i', \quad i = x, \varphi, \tag{18}$$

$$\hat{A}_r\phi = -\int_0^\infty \frac{c_r'}{c_r}\exp\bigl(c_r^2 - c_r'^2\bigr)R_r(c_r' \to c_r)\phi(c_r')\,\mathrm{d}c_r'. \tag{19}$$

Here, the CL scattering kernel $R(\mathbf{c}' \to \mathbf{c})$ has been decomposed into the three parts

$$R(\mathbf{c}' \to \mathbf{c}) = R_x(c'_x \to c_x) R_\varphi(c'_\varphi \to c_\theta) R_r(c'_r \to c_r), \tag{20}$$

which are written down in term of the dimensionless molecular velocity $\mathbf{c}$ as

$$R_i(c'_i \to c_i) = \frac{1}{[\pi\alpha_t(2-\alpha_t)]^{1/2}} \exp\left\{-\frac{[c_i - (1-\alpha_t)c'_i]^2}{\alpha_t(2-\alpha_t)}\right\}, \quad i = x, \varphi, \tag{21}$$

$$R_r(c'_r \to c_r) = -\frac{2c_r}{\alpha_r} \exp\left[-\frac{c_r^2 + (1-\alpha_n)c_r'^2}{\alpha_n}\right] I_0\left(\frac{2\sqrt{1-\alpha_n}\, c_r c'_r}{\alpha_n}\right). \tag{22}$$

It can be shown that the operators $\hat{A}_x$ and $\widehat{A}_\varphi$ satisfy the relations

$$\hat{A}_x c_x = (1-\alpha_t) c_x, \tag{23}$$

$$\hat{A}_x c_x^2 = (1-\alpha_t)^2 c_x^2 + \frac{1}{2}\alpha_t(2-\alpha_t), \tag{24}$$

$$\hat{A}_x c_x^3 = (1-\alpha_t)^3 c_x^3 + \frac{3}{2} c_x \alpha_t(2-\alpha_t)(1-\alpha_t), \tag{25}$$

$$\hat{A}_x \hat{A}_\varphi c_x\big(c_x^2 + c_\theta^2 - 2\big) = (1-\alpha_t)^3 c_x\big(c_x^2 + c_\theta^2 - 2\big). \tag{26}$$

The solution of Eq. (10) can be presented as

$$h(r, \mathbf{c}) = 2\varphi(r, c_r, c_\theta) c_x + \psi(r, c_r, c_\theta) c_x \left(c_x^2 - \frac{3}{2}\right). \tag{27}$$

Then, Eq. (10) is split into two coupled equations for the φ and ψ

$$c_r \frac{\partial\varphi}{\partial r} - \frac{c_\theta}{r}\frac{\partial\varphi}{\partial\theta} = \delta\left[u + \frac{2}{15} q\big(c_p^2 - 1\big) - \varphi\right] - \frac{1}{2}\big[\xi_{\mathrm{P}} + \xi_{\mathrm{T}}\big(c_p^2 - 1\big)\big], \tag{28}$$

$$c_r \frac{\partial\psi}{\partial r} - \frac{c_\theta}{r}\frac{\partial\psi}{\partial\theta} = \delta\left[\frac{4}{15} q - \psi\right] - \xi_{\mathrm{T}}, \tag{29}$$

which are coupled via the moments

$$u(r) = \frac{1}{\pi}\int \exp\big(-c_p^2\big)\varphi(r, c_r, c_\theta)\, \mathrm{d}c_r\, \mathrm{d}c_\theta, \tag{30}$$

$$q(r) = \frac{1}{\pi}\int \exp\big(-c_p^2\big)\left[\varphi(r, c_r, c_\theta)\big(c_p^2 - 1\big) + \frac{3}{4}\psi(r, c_r, c_\theta)\right] \mathrm{d}c_r\, \mathrm{d}c_\theta. \tag{31}$$

Here, the notation

$$c_p^2 = c_r^2 + c_\theta^2 \tag{32}$$

has been introduced.

Substituting (27) into (17) with the help of Eqs. (23)–(26) we obtain the boundary conditions for φ and ψ as

$$\varphi^+ = (1-\alpha_t)\hat{A}_r \varphi^-, \qquad \psi^+ = (1-\alpha_t)^3 \hat{A}_r \psi^-. \tag{33}$$

These conditions mean that at $\alpha_t = 1$ the solution of Eqs. (28) and (29) does not depend on the energy accommodation coefficient α_n and corresponds to that for the diffuse reflection.

The system of kinetic equations for φ and ψ was solved by the discrete velocity method, which was described in details in the previous paper [12] and is omitted here. The numbers of points in both physical and velocity spaces were chosen so as to provide the numerical error of the coefficients G_{P} and G_{T} less than 0.1%.

3. Results and discussions

The numerical calculations were carried out in the range of the momentum accommodation coefficients α_t from 0.25 to 1.75 and in the range of the energy accommodation coefficient α_n from 0 to 1. These ranges of α_t and α_n embrace all physical situations that may occur in practice. The rarefaction parameter δ varied from 0 to 50. The numerical results on G_{P} and G_{T} are presented on Tables 1 and 2, respectively.

Table 1
Poiseuille flow G_{P} vs δ, α_t and α_n

δ	α_t	G_{P}			
		$\alpha_n = 0.25$	0.5	0.75	1
0.0	0.25	7.237	7.122	7.062	7.028
	0.5	3.402	3.358	3.331	3.312
	0.75	2.132	2.118	2.107	2.099
	1.0	1.505	1.505	1.505	1.505
	1.25	1.123	1.132	1.139	1.147
	1.5	0.8385	0.8556	0.8711	0.8865
	1.75	0.5733	0.6038	0.6311	0.6588
0.01	0.25	7.191	7.084	7.029	6.995
	0.5	3.369	3.328	3.302	3.284
	0.75	2.103	2.089	2.079	2.071
	1.0	1.477	1.477	1.477	1.477
	1.25	1.096	1.105	1.112	1.119
	1.5	0.8135	0.8296	0.8445	0.8594
	1.75	0.5507	0.5796	0.6058	0.6325
0.1	0.25	7.015	6.956	6.920	6.897
	0.5	3.267	3.242	3.225	3.212
	0.75	2.024	2.015	2.008	2.003
	1.0	1.409	1.409	1.409	1.409
	1.25	1.035	1.041	1.046	1.051
	1.5	0.7614	0.7722	0.7829	0.7940
	1.75	0.5144	0.5344	0.5535	0.5736
1.0	0.25	6.862	6.858	6.855	6.854
	0.5	3.267	3.265	3.264	3.263
	0.75	2.071	2.070	2.070	2.069
	1.0	1.477	1.477	1.477	1.477
	1.25	1.119	1.120	1.121	1.121
	1.5	0.8709	0.8726	0.8736	0.8742
	1.75	0.6741	0.6770	0.6789	0.6805
5.0	0.25	7.725	7.711	7.699	7.690
	0.5	4.163	4.154	4.147	4.140
	0.75	2.968	2.964	2.960	2.957
	1.0	2.366	2.366	2.366	2.366
	1.25	2.000	2.005	2.008	2.011
	1.5	1.752	1.761	1.767	1.774
	1.75	1.570	1.582	1.592	1.601
10.0	0.25	8.949	8.929	8.911	8.895
	0.5	5.385	5.372	5.360	5.350
	0.75	4.185	4.179	4.173	4.168
	1.0	3.577	3.577	3.577	3.577
	1.25	3.205	3.212	3.217	3.222
	1.5	2.952	2.964	2.975	2.986
	1.75	2.767	2.784	2.800	2.816
50.0	0.25	18.92	18.90	18.87	18.85
	0.5	15.35	15.34	15.32	15.30
	0.75	14.15	14.14	14.13	14.12
	1.0	13.53	13.53	13.53	13.53
	1.25	13.15	13.16	13.17	13.18
	1.5	12.89	12.91	12.93	12.94
	1.75	12.70	12.73	12.75	12.77

Near the hydrodynamic regime ($\delta \gg 1$) the flow rates G_{P} and G_{T} can be expressed as [2]

$$G_{\mathrm{P}} = \frac{\delta}{4} + \sigma_{\mathrm{P}}, \qquad G_{\mathrm{T}} = \frac{\sigma_{\mathrm{T}}}{\delta}, \tag{34}$$

Table 2
Thermal creep G_T vs δ, α_t and α_n

δ	α_t	G_T			
		$\alpha_n = 0.25$	0.5	0.75	1.
0.0	0.25	1.4785	1.1847	1.0328	0.9420
	0.5	1.0255	0.9124	0.8382	0.7864
	0.75	0.8462	0.8084	0.7792	0.7558
	1.0	0.7523	0.7523	0.7523	0.7523
	1.25	0.6854	0.7072	0.7280	0.7488
	1.5	0.6081	0.6461	0.6842	0.7252
	1.75	0.4893	0.5454	0.6022	0.6664
0.01	0.25	1.4206	1.1433	0.9975	0.9094
	0.5	0.9849	0.8769	0.8054	0.7550
	0.75	0.8118	0.7755	0.7473	0.7245
	1.0	0.7210	0.7210	0.7210	0.7210
	1.25	1.0965	1.1047	1.1121	1.1190
	1.5	0.8135	0.8296	0.8445	0.8594
	1.75	0.5507	0.5796	0.6058	0.6325
0.1	0.25	1.1496	0.9705	0.8644	0.7956
	0.5	0.8233	0.7475	0.6934	0.6533
	0.75	0.6908	0.6642	0.6423	0.6241
	1.0	0.6206	0.6206	0.6206	0.6206
	1.25	0.5688	0.5848	0.6008	0.6172
	1.5	0.5046	0.5326	0.5621	0.5945
	1.75	0.4036	0.4449	0.4889	0.5395
1.0	0.25	0.4938	0.4856	0.4787	0.4725
	0.5	0.4282	0.4232	0.4186	0.4144
	0.75	0.4051	0.4029	0.4008	0.3988
	1.0	0.3968	0.3968	0.3968	0.3968
	1.25	0.3894	0.3911	0.3928	0.3948
	1.5	0.3708	0.3740	0.3775	0.3815
	1.75	0.3322	0.3374	0.3431	0.3498
5.0	0.25	0.1596	0.1682	0.1766	0.1847
	0.5	0.1624	0.1679	0.1733	0.1786
	0.75	0.1686	0.1712	0.1739	0.1765
	1.0	0.1762	0.1762	0.1762	0.1762
	1.25	0.1833	0.1810	0.1785	0.1759
	1.5	0.1885	0.1839	0.1790	0.1739
	1.75	0.1900	0.1835	0.1764	0.1689
10.0	0.25	0.0873	0.0930	0.0986	0.1040
	0.5	0.0918	0.0954	0.0990	0.1026
	0.75	0.0968	0.0986	0.1003	0.1021
	1.0	0.1020	0.1020	0.1020	0.1020
	1.25	0.1071	0.1054	0.1037	0.1019
	1.5	0.1114	0.1082	0.1049	0.1014
	1.75	0.1147	0.1101	0.1053	0.1002
50.0	0.25	0.0188	0.0201	0.0215	0.0228
	0.5	0.0202	0.0211	0.0220	0.0229
	0.75	0.0216	0.0220	0.0224	0.0229
	1.0	0.0229	0.0229	0.0229	0.0229
	1.25	0.0241	0.0237	0.0233	0.0228
	1.5	0.0253	0.0245	0.0237	0.0229
	1.75	0.0265	0.0253	0.0241	0.0229

where σ_P and σ_T are the viscous and thermal slip coefficients, respectively, which were calculated on the basis of the CL kernel in the previous paper [4]. Comparing the numerical values of G_P and G_T at $\delta = 50$ with those calculated via the expressions (34) we can see that in the range $\delta \geqslant 50$ these expressions provide the Poiseuille flow G_P with the error less than 1% and the thermal creep G_T with the error less than 3%.

So, the numerical data given in Tables 1 and 2 together with the expressions (34) and numerical data presented in Tables 1 and 3 of the work [4] cover the whole range of the rarefaction parameter δ. From these data one can see that:

(i) The Poiseuille flow G_{P} significantly depends on the accommodation coefficient of tangential momentum α_t, while its dependence on the energy accommodations coefficient α_n is weak.
(ii) In accordance with Eq. (34) the dependence of the Poiseuille flow G_{P} on the gas–surface interaction law disappears in the hydrodynamic limit $\delta \to \infty$.
(iii) The thermal creep G_{T} significantly depends on both accommodation coefficients α_t and α_n in the whole range of the gas rarefaction δ including the hydrodynamic regime ($\delta \to \infty$).
(iv) The dependence of G_{T} on α_t and α_n in the free molecular regime ($\delta = 0$) is quite different from that in the hydrodynamic regime ($\delta = \infty$) given by Eq. (34), namely, for $\delta = 0$ the thermal creep G_{T} reaches its maximum value at $\alpha_t = 0.25$ and $\alpha_n = 0.25$, while for $\delta \to \infty$ the coefficient G_{T} reaches its minimum value at the same α_t and α_n.

The TPD exponent γ can be calculated via G_{P} and G_{T} by (16) if the temperature difference is small. As was shown in the paper [12] the relation (16) can be used at any temperature difference in the free molecular regime ($\delta = 0$). If one calculates the quantity γ on the basis of the Maxwell boundary condition (2) in the free molecular regime, one obtains $\gamma = 0.5$ for any value of the parameter ε. However, the experiment data [5,6] showed that the exponent γ can be lower than 0.5 and for some noble gases the quantity γ is about 0.4. So, the Maxwell kernel (2) cannot explain such a variation of γ. From the data given in Tables 1 and 2 we can see that the quantity γ calculated in the free molecular regime ($\delta = 0$) on the basis of the CL kernel varies in the wide range, namely, from 0.134 at $\alpha_t = 0.25$ and $\alpha_n = 1$ up to 1.01 at $\alpha_t = 1.75$ and $\alpha_n = 1$. This fact shows that the CL kernel provides more physical description of the gas–surface interaction than the Maxwell model (2).

4. Comparison with experiment

The experimental data on the Poiseuille flow G_{P} by Borisov [14] based on measurements of the flow rate of the noble gases through a glass capillary are presented in Fig. 1 by the symbols. The same experimental data and a description of the experimental conditions can be found in the paper [15]. To perform a comparison with these data some additional calculations were carried out for various values of the momentum accommodation coefficient α_t. Since the dependence of G_{P} on the energy accommodation coefficient is weak it was assumed $\alpha_n = 1$ in these calculations. It was found that the experimental data for helium, neon and argon are in a good agreement with the numerical data for the values $\alpha_t = 0.91$, 0.89 and 0.96, respectively. These numerical data are presented in Fig. 1 by the curves.

The experimental data on the TPD exponent γ by Kulev [16] for the noble gases and for a glass capillary are shown in Fig. 2 by the symbols. The same experimental data and a description of the experimental conditions can be found in the work [17]. Numerical data for some values of the momentum accommodation coefficient α_t assuming $\alpha_n = 1$ are also given in Fig. 2 by the curves. One can see that the numerical data corresponding to the values $\alpha_t = 0.9$, 0.82, 0.94 and 1 are in a good agreement with the experimental results for helium, neon, argon and xenon, respectively.

Porodnov et al. [15] reported some experimental data on the mass flow rate through a glass capillary sieve in the free molecular regime ($\delta = 0$). The mass flow rates for several gases are given relatively to the flow rate of xenon. However, from the experimental data on the slip coefficient σ_{P} reported by the same authors [15] and from the data on the TPD [16,17] showed in Fig. 2 we conclude that the interaction of xenon with a glass surface is completely diffuse. Considering this fact the experimental results reported in Table 6 of [15] were recalculated in term of the coefficient G_{P} and presented in the second column of Table 3. In the third column of the same table the corresponding values of the momentum accommodation coefficient α_t calculated under assumption $\alpha_n = 1$ are given.

5. Concluding remarks

The Poiseuille flow G_{P} and the thermal creep G_{T} were calculated in the wide ranges of the accommodation coefficients α_t and α_n of the Cercignani–Lampis scattering kernel and in the whole range of the gas rarefaction δ. These numerical data allow us to calculate the thermomolecular pressure difference (TPD). Unlike the Maxwell boundary condition, the CL kernel provides the variable value of the TPD exponent in the free molecular regime. This fact explains us the different experimental values of the TPD exponent γ obtained for different gases [5,6].

Table 4 contains the values of the momentum accommodation coefficient α_t calculated from various quantities measured in different experiments. The numerical values of α_t corresponding to the experimental results of the slip coefficient σ_{P} were calculated in the previous paper [4]. From these data we can see that the interaction of xenon with a glass surface is characterized

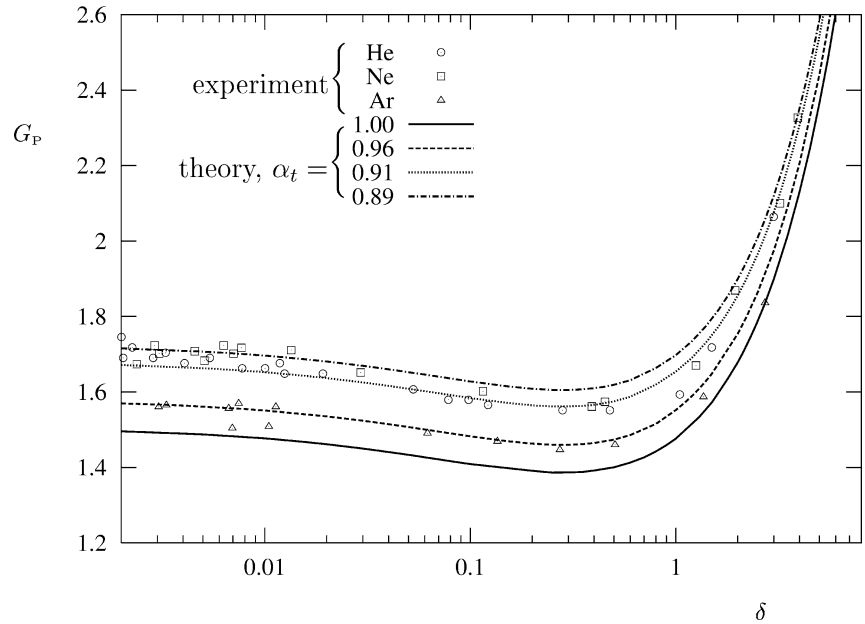

Fig. 1. Poiseuille flow $G_{\rm P}$ vs rarefaction parameter δ: symbols – experimental data for a glass capillary [14,15]; curves – present numerical results at $\alpha_n = 1$.

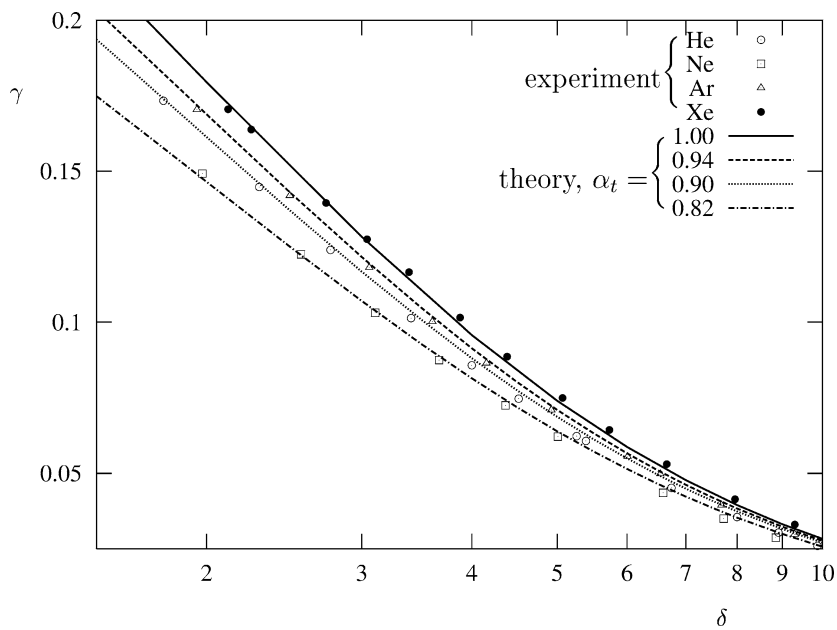

Fig. 2. TPD exponent γ vs rarefaction parameter δ: symbols – experimental data for a glass capillary [16,17]; curves – present numerical results at $\alpha_n = 1$.

as the perfect accommodation, i.e., $\alpha_t = 1$. The experimental data by Podgurski and Davis [5] on the TPD confirm this conclusion. The interaction of krypton with a glass surface is close to the perfect accommodation. Argon has a small deviation from the perfect accommodation, while helium and neon represent a significant deviation from the diffuse interaction. Table 4 shows that the values of the momentum accommodation coefficient α_t obtained form different experiments are close to each other. So, the CL kernel (3) describes the gas–surface interaction more physically than the Maxwell boundary condition (2).

Table 3
Experimental values [15] of the mass flow rate G_P through a glass capillary sieve in the free molecular regime ($\delta = 0$) and the corresponding values of the momentum accommodation coefficient α_t assuming $\alpha_n = 1$

Gas	G_P	α_t
He	1.684 ± 0.008	0.908 ± 0.003
Ne	1.717 ± 0.008	0.893 ± 0.003
Ar	1.560 ± 0.015	0.969 ± 0.007
Kr	1.532 ± 0.01	0.985 ± 0.007
Xe	1.505	1

Table 4
Values of α_t obtained from different experiments for glass capillary

Measured quantity	α_t				
	He	Ne	Ar	Kr	Xe
G_P at $\delta = 0$, [15]	0.908	0.893	0.969	0.985	1
G_P at $\delta \sim 1$, [14,15]	0.91	0.89	0.96	–	–
σ_P, [15]	0.882*	0.849	0.917	0.998	1.016
γ, [16,17]	0.90	0.82	0.94	–	1

* The values of α_t based on σ_P were obtained in [4].

The quantities G_P, γ and σ_P used in the present work and in the previous paper [4] to calculate the momentum accommodation coefficient α_t do not allows us to obtain the value of the energy accommodation coefficient α_n, because these quantities are not sensitive to α_n. To estimate this accommodation coefficient a problem related to a heat transfer between gas and surface should be solved on the basis of the CL kernel. Then a comparison of these results with corresponding experimental data will allow us to obtain the values of α_n. This is a task for our future work.

It should be noted that to carry out a further investigation of the scattering kernel we need more accurate and more reliable experimental data on the Poiseuille flow, thermal creep, thermomolecular pressure difference, velocity slip and temperature jump coefficients.

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